

# Corrections

## Quantum Mechanics: Concepts and Applications

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*(Modified last on September 30, 2010)*

**Note:** This list of corrections pertains to: N. Zettili, Quantum Mechanics: Concepts and Applications, (Chichester: John-Wiley, 2009); 2nd edition, ISBN: 978-0-470-02678-6 (Hardcover), 978-0-470-02679-3 (Paperback).

### Chapter 2

- Page 151, replace Eq. (2.471) by

$$\begin{aligned}\hat{H}^2 &= \alpha^2 (|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|) (|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|) \\ &= \alpha^2 (|\phi_1\rangle\langle\phi_1| + |\phi_2\rangle\langle\phi_2|),\end{aligned}$$

- Page 8, Page 151, replace Eq. (2.472) by

$$\begin{aligned}(\alpha^{-2}\hat{H}^2)^2 &= (|\phi_1\rangle\langle\phi_1| + |\phi_2\rangle\langle\phi_2|) (|\phi_1\rangle\langle\phi_1| + |\phi_2\rangle\langle\phi_2|) \\ &= |\phi_1\rangle\langle\phi_1| + |\phi_2\rangle\langle\phi_2| = \alpha^{-2}\hat{H}^2.\end{aligned}$$

### Chapter 3

- Replace Problem 3.5 and its solution on Page 196 by

#### Problem 3.5

Assuming that the system of Problem 3.4 is initially in the state  $|\phi_3\rangle$ , what values for the energy and the observable  $A$  will be obtained if we measure: (i)  $H$  first then  $A$ , (ii)  $A$  first then  $H$ ?

(b) Compare the results obtained in (i) and (ii) and infer whether  $\hat{H}$  and  $\hat{A}$  are compatible. Calculate  $[\hat{H}, \hat{A}]|\phi_3\rangle$ .

(c) Consider now an other operator  $\hat{B}$  whose action on  $|\phi_n\rangle$  is defined by  $\hat{B}|\phi_n\rangle = nb_0|\phi_{n+1}\rangle$ . Repeat Questions (a) and (b) for the operator  $\hat{B}$ .

#### Solution

(a) (i) The measurement of  $H$  first then  $A$  is represented by  $\hat{A}\hat{H}|\phi_3\rangle$ . Using the relations  $\hat{H}|\phi_n\rangle = n^2\mathcal{E}_0|\phi_n\rangle$  and  $\hat{A}|\phi_n\rangle = (n+1)a_0|\phi_n\rangle$ , we have

$$\hat{A}\hat{H}|\phi_3\rangle = 9\mathcal{E}_0\hat{A}|\phi_3\rangle = 36\mathcal{E}_0a_0|\phi_3\rangle. \quad (1)$$

(ii) Measuring  $A$  first and then  $H$ , we will obtain

$$\hat{H}\hat{A}|\phi_3\rangle = 4a_0\hat{H}|\phi_3\rangle = 36\mathcal{E}_0a_0|\phi_3\rangle. \quad (2)$$

(b) Equations (1) and (2) show that the actions of  $\hat{A}\hat{H}$  and  $\hat{H}\hat{A}$  yield the same result. This means that  $\hat{H}$  and  $\hat{A}$  commute; hence they are compatible. We can thus write

$$[\hat{H}, \hat{A}]|\phi_3\rangle = (36 - 36)\mathcal{E}_0a_0|\phi_3\rangle = 0. \quad (3)$$

(c) (i) The measurement of  $H$  first then  $B$  is represented by  $\hat{B}\hat{H}|\phi_3\rangle$ . Using the relations  $\hat{H}|\phi_n\rangle = n^2\mathcal{E}_0|\phi_n\rangle$  and  $\hat{B}|\phi_n\rangle = nb_0|\phi_{n+1}\rangle$ , we have

$$\hat{B}\hat{H}|\phi_3\rangle = 9\mathcal{E}_0\hat{B}|\phi_3\rangle = 27\mathcal{E}_0b_0|\phi_4\rangle. \quad (4)$$

(ii) Measuring  $B$  first and then  $H$ , we will obtain

$$\hat{H}\hat{B}|\phi_3\rangle = 3b_0\hat{H}|\phi_4\rangle = 48\mathcal{E}_0b_0|\phi_4\rangle. \quad (5)$$

Equations (4) and (5) show that the actions of  $\hat{B}\hat{H}$  and  $\hat{H}\hat{B}$  yield different results. This means that  $\hat{H}$  and  $\hat{B}$  do not commute; hence they are not compatible. We can thus write

$$[\hat{H}, \hat{B}]|\phi_3\rangle = (48 - 27)\mathcal{E}_0b_0|\phi_4\rangle = 17\mathcal{E}_0b_0|\phi_4\rangle. \quad (6)$$

- Replace the first two lines of Problem 3.11 on Page 204 by:

Consider a system whose initial state  $|\psi(0)\rangle$  and Hamiltonian are given by

$$|\psi(0)\rangle = \frac{1}{5} \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}, \quad H = \epsilon \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 5 \\ 0 & 5 & 0 \end{pmatrix},$$

where  $\epsilon$  has the dimensions of an energy.

- Replace the third line from the bottom of Page 204 by:

A measurement of the energy yields the values  $E_1 = -5\epsilon$ ,  $E_2 = 3\epsilon$ ,  $E_3 = 5\epsilon$ ; the

- Replace Eq. (3.222) on Page 205 by:

$$|\psi(t)\rangle = \frac{2\sqrt{2}}{5}e^{-iE_1t/\hbar}|\phi_1\rangle + \frac{3}{5}e^{-iE_2t/\hbar}|\phi_2\rangle + \frac{2\sqrt{2}}{5}e^{-iE_3t/\hbar}|\phi_3\rangle = \frac{1}{5} \begin{pmatrix} 3e^{-3i\epsilon t/\hbar} \\ -4i \sin(5\epsilon t/\hbar) \\ 4 \cos(5\epsilon t/\hbar) \end{pmatrix}.$$

- Replace Eq. (3.223) on Page 205 by:

$$\begin{aligned} E(0) &= \langle \psi(0) | \hat{H} | \psi(0) \rangle = \frac{8}{25} \langle \phi_1 | \hat{H} | \phi_1 \rangle + \frac{9}{25} \langle \phi_2 | \hat{H} | \phi_2 \rangle + \frac{8}{25} \langle \phi_3 | \hat{H} | \phi_3 \rangle \\ &= \frac{8}{25}(-5)\epsilon + \frac{9}{25}(3)\epsilon + \frac{8}{25}(5)\epsilon = \frac{27}{25}\epsilon. \end{aligned}$$

- Replace Eq. (3.224) on Page 205 by:

$$E(0) = \langle \psi(0) | \hat{H} | \psi(0) \rangle = \frac{\epsilon}{25} \begin{pmatrix} 3 & 0 & 4 \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 5 \\ 0 & 5 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} = \frac{27}{25} \epsilon.$$

- Replace Eq. (3.225) on Page 205 by:

$$E(0) = \sum_{n=1}^2 P(E_n) E_n = \frac{8}{25}(-5)\epsilon + \frac{9}{25}(3)\epsilon + \frac{8}{25}(5)\epsilon = \frac{27}{25} \epsilon.$$

- Replace Eq. (3.226) on Page 205 by:

$$\begin{aligned} E(t) &= \langle \psi(t) | \hat{H} | \psi(t) \rangle = \frac{8}{25} e^{iE_1 t/\hbar} e^{-iE_1 t/\hbar} \langle \phi_1 | \hat{H} | \phi_1 \rangle + \frac{9}{25} e^{iE_2 t/\hbar} e^{-iE_2 t/\hbar} \langle \phi_2 | \hat{H} | \phi_2 \rangle \\ &\quad + \frac{8}{25} e^{iE_3 t/\hbar} e^{-iE_3 t/\hbar} \langle \phi_3 | \hat{H} | \phi_3 \rangle = \frac{8}{25}(-5)\epsilon + \frac{9}{25}(3)\epsilon + \frac{8}{25}(5)\epsilon = \frac{27}{25} \epsilon = E(0). \end{aligned}$$