Let's talk about taking another trip. You lost your debit card and there isn't time to get another one before you leave, so you're going to just use cash this time. So you leave town with $428 in your wallet. Just like before we can come up with a quantity called "Estimated budget" (E) if there is one day on which you know both how much you have spent and how much is in your wallet. So the day you leave town we have:

\[
\begin{align*}
\text{Spent} & \quad + \quad \text{Cash} = E \\
\$0 & \quad + \quad \$428 = \$428 \\
\end{align*}
\]

So $428 is your estimated budget. On any day after that, as long as you know how much is left in your wallet, you can find how much you've spent. For example, on the third day of your trip you have $359 in your wallet. So you can find how much you've spent by

\[
\begin{align*}
E & \quad - \quad \text{Cash} = \text{Spent} \\
\$428 & \quad - \quad \$359 = \$69 \\
\end{align*}
\]

Or on that third day of the trip you could have added up your receipts, found that you'd spent $69 and did a similar calculation to find how much money was left.

\[
\begin{align*}
E & \quad - \quad \text{Spent} = \text{Cash} \\
\$428 & \quad - \quad \$69 = \$359 \\
\end{align*}
\]

On the fifth day of your trip you are staying with friends of a friend. You had planned on doing some sightseeing that day, but they needed to ask a favor. Comcast is supposed to come by and install cable and someone needs to be there. However they have to go to classes and so they asked if you could pay them $50 to wait around until the cable guy comes. You agree. They have an Xbox and some games you've never played before so it's a pretty easy morning's work for you (definitely no sweating involved).

But let's think about what that $50 does to your money calculations. Basically you now have two different estimated budgets (E) – one before you "worked" and one after. They are related by:

\[
E_{\text{before}} + \text{"Work"} = E_{\text{after}} \\
\$428 + \$50 = \$478 \\
\]

Any money calculations you do after that will have to use the new Estimated budget, $E_{\text{after}}$.

One the sixth day of your trip you count up the money in your wallet and find that you have $312 left. How much have you spent at that point?

\[
\begin{align*}
\text{Spent} & = E_{\text{after}} - \text{Cash} \\
\text{Spent} & = \$478 - \$312 \\
\text{Spent} & = \$166 \\
\end{align*}
\]

One the seventh day of your trip you add up your receipts and found that you've spent $266. How much money would you find in your wallet?

\[
\begin{align*}
\text{Cash} & = E_{\text{after}} - \text{Spent} \\
\text{Cash} & = \$478 - \$266 \\
\text{Cash} & = \$212 \\
\end{align*}
\]
As we saw in the previous worksheet, in physics we don't have receipts to count, but anytime we want to know how much the object has "spent" on Kinetic Energy (KE) we can figure it out as long as we know how fast it is moving using this equation:

\[ \text{KE} = \frac{1}{2}mv^2 \]  
**Equation 1**

We also don't have a wallet full of money but we do have Gravitational Potential Energy (PE). As long as we know “how high up” (h) then we can figure out PE using this equation:

\[ \text{PE} = mgh \]

**Equation 2**

We also don't have an estimated budget (E), but we do have something called the Total Mechanical Energy (E) which works the same way. Putting this in mathematics we have:

\[ \text{(KE) + (PE) = E} \]

**Equation 3**

This is just like:

\[ \text{Spent + Cash = E} \]

Work doesn't mean getting money in physics, it means getting energy. The equation we use to figure out how much energy is given to the object (how much "work is done upon the object") using this equation:

\[ \text{Work} = F \Delta x \cos(\theta) \]

**Equation 4**

Our situation this time is shown in the diagram below. A cart with a motor and propeller starts out at the top of a tilted ramp. Initially the cart starts off at \( h=2.50 \text{m} \) (we'll take \( h=0 \) to be floor level). The cart, motor and propeller have a total mass of 850g and the fan can push the cart with a force of 15.0N. We also give it a push so that it has an initial speed down the ramp of 0.75m/s, but the cart begins with the fan initially turned off.
What is the kinetic energy at position A?

\[ KE_A = \frac{1}{2} m v_A^2 \]
\[ KE_A = \frac{1}{2} \times (0.850 \text{ kg}) \times (0.75 \text{ m/s})^2 \]
\[ KE_A = 0.239 \text{ J} \]

What is the potential energy at position A?

\[ PE_A = mgh_a \]
\[ PE_A = (0.850 \text{ kg}) \times (9.8 \text{ m/s}^2) \times (2.50 \text{ m}) \]
\[ PE_A = 20.8 \text{ J} \]

Since we know both these quantities in that initial position we can calculate the Total mechanical energy \( E_1 \) by adding KE and PE. What is \( E_1 \)?

\[ E_1 = PE_A + KE_A \]
\[ E_1 = 20.8 \text{ J} + 0.239 \text{ J} \]
\[ E_1 = 21.04 \text{ J} \]

At position B the height is now \( h = 2.25 \text{ m} \). What is PE at position B?

\[ PE_B = mgh_b \]
\[ PE_B = (0.850 \text{ kg}) \times (9.8 \text{ m/s}^2) \times (2.25 \text{ m}) \]
\[ PE_B = 18.74 \text{ J} \]

Use \( E_1 \) and the PE at position B to calculate KE at position B.

\[ KE_B = E_1 - PE_B \]
\[ KE_B = 21.04 \text{ J} - 18.74 \text{ J} \]
\[ KE_B = 2.30 \text{ J} \]

Use KE at position B and Equation 1 to calculate the velocity at position B.

\[ KE_B = \frac{1}{2} m v_B^2 \]
\[ 2.30 \text{ J} = \frac{1}{2} \times (0.850 \text{ kg}) \times v_B^2 \]
\[ v_B = 2.33 \text{ m/s} \]

The velocity at position C is 4.00 m/s. Use this to calculate the height \( h \) at position C.

\[ KE_C = \frac{1}{2} m v_C^2 \]
\[ KE_C = \frac{1}{2} \times (0.850 \text{ kg}) \times (4.00 \text{ m/s})^2 \]
\[ KE_C = 6.80 \text{ J} \]

There is a switch mounted in the track at position C that switches on the fan. The fan pushes the cart as it rolls through the next 75 cm of the track. When it reaches position D another switch mounted in the track turns the fan back off.

How much work did the fan do on the cart?

\[ \text{Work} = F \cdot \Delta x \cdot \cos \theta \]
\[ \text{Work} = (15.0 \text{ N})(0.75 \text{ m}) \cos (0^\circ) \]
\[ \text{Work} = 11.25 \text{ J} \]

What is the new total mechanical energy \( E_2 \) of the cart?

\[ E_2 = E_1 + \text{Work} \]
\[ E_2 = 21.04 \text{ J} + 11.25 \text{ J} \]
\[ E_2 = 32.29 \text{ J} \]

Given that the height at position D is \( h = 1.75 \text{ m} \), what is PE at position D?

\[ PE_D = mgh \]
\[ PE_D = (0.850 \text{ kg}) \times (9.8 \text{ m/s}^2) \times (1.75 \text{ m}) \]
\[ PE_D = 14.58 \text{ J} \]
Use $E_2$ and the PE at position D to calculate the speed of the cart at position D.

\[ KE_0 = E_2 - PE_0 \]
\[ KE_0 = 32.29 J - 14.58 J \]
\[ KE_0 = 17.71 J \]

\[ KE_0 = \frac{1}{2} m v_0^2 \]
\[ 17.71 J = \frac{1}{2} (.850 kg) v_0^2 \]
\[ v_0 = \sqrt{\frac{2(17.71 J)}{.850 kg}} \]
\[ v_0 = 6.45 m/s \]

Given that at position F the speed is 7.5 m/s, calculate the height $h$ of position F.

\[ KE_F = \frac{1}{2} m v_F^2 \]
\[ KE_F = \frac{1}{2} (.850 kg)(7.5 m/s)^2 \]
\[ KE_F = 23.91 J \]

\[ PE_F = E_2 - KE_F \]
\[ PE_F = 32.29 J - 23.91 J \]
\[ PE_F = 8.38 J \]

\[ PE_F = m g h_F \]
\[ 8.38 J = (.850 kg)(9.8 m/s^2) h_F \]
\[ h_F = 1.01 m \]

After position F there is a 1.0 m length of the track that has been damaged and the rough surface exerts a frictional drag of 16.0 N on the cart. This will cause the cart to lose energy (the same as if someone shorted you some money when they gave you your change in a shop during your trip). How much work does friction do on the cart? (Hint: it's taking energy from the cart so this should be a negative number).

\[ \text{Work} = F \cdot \Delta x \cdot \cos \theta \]
\[ \text{Work} = -(16.0 N)(1 m) \cos 180^\circ \]
\[ \text{Work} = -16.0 N \]

What is the new total mechanical energy $E_3$?

\[ E_3 = E_2 + \text{Work} \]
\[ E_3 = 32.29 J + (-16.0 J) \]
\[ E_3 = 16.29 J \]

Use $E_3$ to calculate the speed of the cart at the bottom of the ramp ($h=0$).

\[ E_3 - PE_{bottom} = KE_{bottom} \]
\[ 16.29 J - mg (0) = KE_{bottom} \]
\[ KE_{bottom} = 16.29 J \]

\[ KE_{bottom} = \frac{1}{2} m v_{bottom}^2 \]
\[ 16.29 J = \frac{1}{2} (.850 kg) v_{bottom}^2 \]
\[ v_{bottom} = \sqrt{\frac{2(16.29 J)}{.850 kg}} \]
\[ v_{bottom} = 6.19 m/s \]