Problem #1

See Important Problems 5

Problem #2

Note: "h₂" in the diagram, in the homework should say "h₄", and h₄ = 10m

Here is the corrected diagram:

We are told that the rollercoaster starts at position "1" with vᵢ = 0.
We are to find the speeds at positions 2, 3, and 4.

We know both height and speed at position 2 - so we can use this to calculate the total mechanical energy E.

\[ E = KE₁ + PE₁ \]
\[ E = \frac{1}{2} m v₁² + m g hᵢ \]
\[ E = \frac{1}{2} m (0)² + m (9.8 \text{m/s}²)(35 \text{m}) \]
\[ E = m (9.8 \text{m/s}²)(35 \text{m}) \]

This will be the E that we will use for the rest of the problem.

For position 2:

\[ KE₂ + PE₂ = E \]
\[ \frac{1}{2} m v₂² + m g h₂ = m (9.8 \text{m/s}²)(35 \text{m}) \]
\[ \frac{1}{2} v₂² + 9.8(35) = (9.8 \text{m/s}²)(35 \text{m}) \]
\[ \frac{1}{2} v₂² = (9.8 \text{m/s}²)(35 \text{m}) \]
\[ v₂ = 26.2 \text{ m/s} \]
Problem 2 - 2

For position 3

\[ KE_3 + PE_3 = E \]
\[ \frac{1}{2} m v_3^2 + mgh_3 = m(9.8 \text{ m/s}^2)(35 \text{ m}) \]
\[ \frac{1}{2} v_3^2 + (9.8 \text{ m/s}^2)(24 \text{ m}) = (9.8 \text{ m/s}^2)(35 \text{ m}) \]

\[ v_3 = 14.7 \text{ m/s} \]

For position 4

\[ KE_4 + PE_4 = E \]
\[ \frac{1}{2} m v_4^2 + mgh_4 = m(9.8 \text{ m/s}^2)(35 \text{ m}) \]
\[ \frac{1}{2} v_4^2 + (9.8 \text{ m/s}^2)(10 \text{ m}) = (9.8 \text{ m/s}^2)(35 \text{ m}) \]

\[ v_4 = 22.1 \text{ m/s} \]

Problem 3

A 0.20 kg ball is thrown with a speed of 14 m/s at an angle of 33°.

a) What is its speed at the highest point?

b) How high does it go? (Use the energy approach and ignore air resistance).

A) We just need to use our "old methods" (from before we learned the energy approach). So we know that at the top, \( v_y = 0 \) (momentarily). We also know that \( v_x \) always stays the same (no acceleration in the horizontal direction), so at the top the speed is just \( v_x \). And

\[ v_x = |v| \cos \theta; \]
\[ v_x = (14 \text{ m/s}) \cos (33^\circ) \]

\[ v_x = 11.7 \text{ m/s} \]
b) They specifically asked us to use the energy approach. So we need to figure out the total mechanical energy: $h_f = ?$

\[ E = KE_i + PE_i \]

\[ E = \frac{1}{2} m v_i^2 + mgh_i \]

\[ h_i = 0 \]

\[ E = \frac{1}{2} (0.2\text{kg}) (14\text{m/s})^2 + (0.2\text{kg}) (9.8\text{m/s}^2) (0\text{m}) \]

\[ E = 19.6\text{J} \]

They specifically said that we can ignore friction due to the air — so we won’t lose any energy so $E$ at the highest point we still have $E = 19.6\text{J}$. So

\[ E = KE_f + PE_f \]

\[ E = \frac{1}{2} m v_f^2 + mgh_f \]

\[ 19.6\text{J} = \frac{1}{2} (0.2\text{kg}) (11.74\text{m/s})^2 + (0.2\text{kg}) (9.8\text{m/s}^2) h_f \]

\[ h_f = 2.97\text{m} \]
Problem 4

A novice skier, starting from rest, slides down a frictionless 33.0° incline where vertical height is 130m. How fast is she going when she reaches the bottom?

While it doesn't explicitly say so, we are to assume that there is no friction in this problem.

\[ V_i = 0 \]
\[ h_i = 130m \]
\[ V_f = ? \]
\[ h_f = 0 \]

\[ E = KE_i + PE_i \]
\[ E = \frac{1}{2} m V_i^2 + mgh_i \]
\[ E = \frac{1}{2} m (0m/s)^2 + mg(130m) \]
\[ E = mg(130m) \]

And at the end, we have

\[ E = KE_f + PE_f \]
\[ E = \frac{1}{2} m V_f^2 + mgh_f \]
\[ E = \frac{1}{2} m V_f^2 + mg(0m) \]
\[ E = \frac{1}{2} m V_f^2 \]

Since the initial and final \( E \) are the same, we have

\[ E_i = E_f \]

\[ mg(130m) = \frac{1}{2} m V_f^2 \]

\[ (2.8 m/s^2)(130m) = \frac{1}{2} V_f^2 \]

\[ V_f = 50.5 m/s \]
Problem 5

Jane, looking for Tarzan, is running at top speed (7.0 m/s) and grabs a vine hanging vertically from a tall tree in the jungle. How high can she swing upward?

Jane is basically a pendulum in this problem — but they don't ask us anything about her final angle. All they ask is the final height.

\[ v_i = 0 \text{ m/s} \]
\[ h_i = 0 \text{ m} \]
\[ v_f = 7 \text{ m/s} \]

So:

\[ E_i = KE_i + PE_i \]
\[ E_i = \frac{1}{2} m v_i^2 + mg h_i \]
\[ E_i = \frac{1}{2} m (7.0 \text{ m/s})^2 + m g (0) \]

She swings up until she stops moving. So at the end

\[ E_f = KE_f + PE_f \]
\[ E_f = \frac{1}{2} m v_f^2 + mg h_f \]
\[ E_f = \frac{1}{2} m (0 \text{ m/s})^2 + m g h_f \]
\[ E_f = m (9.8 \text{ m/s}^2) h_f \]
\[ E_i = E_f \]

\[ \frac{1}{2} m (7.0 \text{ m/s})^2 = m (9.8 \text{ m/s}^2) h_f \]

\[ h_f = 2.5 \text{ m} \]
Problem 6

In the high jump, the kinetic energy of an athlete is transformed into gravitational potential energy without the aid of a pole. With what minimum speed must the athlete leave the ground in order to lift his center of mass 1.55 m and cross the bar with a speed of 0.72 m/s?

Problem 6 is basically Problem 3 “done backwards.” We know both the height, hf, and the speed, vfp at the highest point. But all we know about the initial situation is that h1 = 0. We’re supposed to figure out what v1 = ?

\[ E_i = KE_i + PE_i \]
\[ E_i = \frac{1}{2} m v_i^2 + m g h_i \]
\[ E_i = \frac{1}{2} m v_i^2 + m(9.8 \text{ m/s}^2) \]
\[ E_i = \frac{1}{2} m v_i^2 \]

And

\[ E_f = \frac{1}{2} m v_f^2 + m g h_f \]
\[ E_f = \frac{1}{2} m (0.72 \text{ m/s})^2 + m (9.8 \text{ m/s}^2)(1.55 \text{ m}) \]

Since nothing gives or takes energy (no fan is pushing him, and friction doesn’t steal energy) then

\[ E_i = E_f \]
\[ \frac{1}{2} m v_i^2 = \frac{1}{2} m (0.72 \text{ m/s})^2 + m (9.8 \text{ m/s}^2)(1.55 \text{ m}) \]

\[ v_i = 5.56 \text{ m/s} \]