Experiment 2: Springs and Oscillations

2.B. Simple Harmonic Motion

This week you will observe the motion of a mass oscillating on a vertical spring and compare your observations with an analytical prediction and a computational model.

Equipment/supplies provided:

- Sonic ranger, interface box, and computer.
- Spring, wide hanger, additional masses.
- Reinforced pole for hanging spring from

Students are responsible to have:

- Laboratory manual with worksheet pages for recording observations and data.

Experimental Tasks:

1. Measure the characteristics of the mass-spring system.
   1.1. Select a spring and record its identification number on your laboratory worksheet. (You will use the same spring this week and next.) Measure and record basic characteristics of the spring such as its mass and length. See worksheet used spring A.
   1.2. Make measurements that will allow you to determine the spring constant $k$ of your spring and the effective position $y_0$ of the system when the spring is unstretched. Document your results and methods in your lab worksheet. You can measure the position of the mass either manually with a meter stick or using a sonic ranger and the Physics Lab Assistant software. Either way, be consistent with your measurement technique. Be sure to obtain uncertainties on your values for $k$ and $y_0$. Use data on worksheet to find $k$ and $y_0$.
   1.3. Check the accuracy of your results by calculating the predicted position with a mass you didn’t use when finding $k$ and $y_0$, and then test your prediction.

2. Measure the position versus time as the mass-spring system is oscillating and find the oscillation frequency.
   2.1. Set up Physics Lab Assistant to measure the position of the mass attached to the spring, placing the sonic ranger on the ground below the mass, with the upward direction being “positive” and the downward direction being “negative.” Ensure that the sonic ranger is set on its narrow beam setting. Check the Timing Parameters to make sure that the Ranger Period is set to 25 ms. This will cause the sonic ranger to measure position once each 25 milliseconds of elapsed time.

   Note: You may be able to obtain a better position calibration by putting the sonic ranger on the air track, using a glider as a target, and calibrating to the position on the air track scale. If you do this, in order to more easily compare your measurements to the simulation we will do later, place the sonic ranger on the air track so that it is “looking” in the positive direction on your number line.

   Used 0.45 kg total including mass hanger

   2.2. Choose a mass and secure it to the mass hanger on the end of the spring to ensure there is no chance for it to slide off and damage the sonic ranger below it. Set the spring-mass system into motion and record data. If your data is not a smooth sine wave, make adjustments to obtain one; consult with an instructor if needed to collect a smooth set of data.

   This data is saved as "E2B-SHM.exp"
2.3. Using the cursors on the waveform plots determine the period \( T \pm \Delta T \), or time for one oscillation, of your mass-spring system. It is probably reasonable to assume that your uncertainty in identifying the best locations for the cursors is no more than 1 sonic ranger period for each cursor. So, to obtain the best estimate of the period and its uncertainty you should use your cursors to measure the time for several \( N \) periods of the motion. Then your period would be this time divided by \( N \) and the uncertainty will be \( 50 \text{ ms} \) (2 sonic ranger periods) divided by \( N \).

2.4. As a second method for finding the period (and as practice for next week where you will have to use this technique), export your data to a tab-delimited text file and use Physics Data Assistant to fit the data to a sinusoidal model described by

\[
y(t) = y_0 + A \cos(\omega t + \phi).
\]

Since this is a non-linear model, you will have to make rough estimates for the fit parameters \( y_0, A, \omega \) and \( \phi \). You can use the Show Test Fit Using Guesses checkbox to refine your guesses for the fit parameters.

Once you have the best fit values for the parameters calculate a value for the period (and its standard error uncertainty) using the oscillation frequency \( \omega \pm \Delta \omega \), the relation \( T = 2\pi /\omega \), and propagation of uncertainty to find \( \Delta T \).

3. **Model the oscillation of the system using VPython.**

3.1. Using the Chrome browser, navigate to [physics.wku.edu/up/phys256/](http://physics.wku.edu/up/phys256/) and copy the starter code for the mass on a spring model into a new program in your Glowscript account. The visualization of the mass and spring is complete in the starter code, but you will need to add the relevant physics as described in the following steps.

3.2. Follow the instructions provided within the starter code to add attributes to the mass and spring including mass, spring constant, and initial position and velocity/momentum.

3.3. Inside the loop, add lines specifying the force on the mass, updating the momentum and position of the mass, and the position of the end of the spring attached.

3.4. Run your program and determine the period of the mass-spring system predicted by your model. Record your result on your laboratory worksheet.

4. **Compare your experimental, simulated, and analytical values for the period of oscillation.**

4.1. When we study a mass-spring system in a textbook we predict that the period of oscillation should be related to the spring constant and mass by the relationship

\[
T = 2\pi \sqrt{\frac{m}{k}}
\]

Compute this analytical prediction for the period using the mass attached to your spring and the spring constant.

4.2. Compare your measured value for the period (either from step 2.3 where you estimated \( T \) using cursors on your graph or from 2.4 where you performed a sinusoidal fit) to the analytical prediction. Do these values agree? How about the analytical model and the simulation? If you have done the above steps well, it is quite likely that you will find that your experimental value **does not** agree within uncertainties with either the analytical model or the simulation.
While this could be due to errors in conducting the experiment or setting up your program, it could also be because the analytical model and simulation do not take into account a relevant aspect of the physical situation. Think about what possible physical factors, not included in your simple physical model, might affect the period of the oscillator.

**Hint:** Recall your prelab question that used energy principles to determine an effective mass of the oscillating spring. This problem showed that if the free end of the spring is moving with a speed $V$, then the kinetic energy associated with the spring motion is

$$K = \frac{1}{6} m_s V^2 = \frac{1}{2} \left( \frac{m_s}{3} \right) V^2.\]$$

This model predicts that you can consider the effective mass of the spring to be equal to one-third of the spring’s total mass. Try adding $\frac{1}{3}$ the mass of the spring to your hanging mass in the simulation and see if you get a result that agrees with your experiment better.

4.3. Our springs are not perfectly cylindrical, so the $\frac{1}{3}$ value doesn’t quite hold. Find the fraction of the spring mass that does give agreement between the measured and predicted periods. Do this by setting your experimentally measured period equal to

$$T = 2\pi \sqrt{ \frac{m + f m_s}{k}}$$

where $m$ is the mass hanging from the spring, $m_s$ is the mass of the spring itself, $k$ is the spring constant, and $f$ is a number between 0 and 1 representing the fraction of the spring mass that must be included in the model. How does your value of $f$ compare to the $\frac{1}{3}$ ratio predicted by the energy model?

For Next Week

- Prepare a draft report that consists of a brief description of your model, your measurements, experimental and computational graphs, and important calculations, along with identifying text.
- Complete the pre-lab questions for next week.
### Experiment 2: Springs and Oscillations

#### Part 2B – Simple Harmonic Motion

<table>
<thead>
<tr>
<th>Name</th>
<th>Partner</th>
<th>Date</th>
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<tbody>
<tr>
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<table>
<thead>
<tr>
<th>Spring ID Number:</th>
<th>Spring Mass ( (m_0) ):</th>
<th>Spring Length ( (L_0) ):</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( 0.0915 \text{ kg} )</td>
<td>( 0.18 \text{ m} )</td>
</tr>
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</table>

#### Measurements for characterizing spring

<table>
<thead>
<tr>
<th>Mass ( (kg) )</th>
<th>Force ( (N) )</th>
<th>Position ( (m) )</th>
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<tbody>
<tr>
<td>0.25</td>
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<td>0.55</td>
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#### Results

- Equilibrium position: \( y_0 \pm \Delta y_0 = \) __________
- Spring Constant: \( k \pm \Delta k = \) __________
- Verification
  - New Trial Mass: \( m = 0.45 \text{ kg} \)
  - Predicted Position: \( y \pm \Delta y = \) __________
  - Measured Position: \( y \pm \Delta y = \) \( 0.655 \pm 0.005 \text{ m} \)

Does your predicted position of the mass agree within uncertainties with your measured position?

<table>
<thead>
<tr>
<th>Sonic Ranger Gain: ( 172.941 \text{ m/s} )</th>
<th>Sonic Ranger Offset: ( -0.43644 \text{ m} )</th>
<th>Hanging Mass ( (m) ): ( 0.45 \text{ kg} )</th>
</tr>
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</table>

Period of oscillating mass by using cursors and counting oscillations \( (T \pm \Delta T) \):

Fit of data to \( y(t) = y_0 + A \cos(\omega t + \phi) \):

- Equilibrium position: \( y_0 \pm \Delta y_0 = \) __________
- Amplitude: \( A \pm \Delta A = \) __________
- Angular frequency: \( \omega \pm \Delta \omega = \) __________
- Phase angle: \( \phi \pm \Delta \phi = \) __________

Calculation of period from best fit:

- Period: \( T \pm \Delta T = \) __________

Theoretical period based on spring characteristics:

\[
T = 2\pi \sqrt{\frac{m}{k}}
\]
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VPython simulation of position versus time:
How does the period from the simulation compare to the theoretical period?

What additional physical factor(s), not included in the basic theoretical model we used, might affect the period of the oscillator?

What is the value of the effective mass needed to get the theory to match the measured period?

What is the ratio of the ("extra" mass) / (spring mass)?

Write a paragraph that summarizes these results and describes how the ratio you found compares to the ratio found in the energy model from your pre-lab questions?

Attachments:
- Annotated graph showing measured position vs time for simple harmonic oscillator, with best fit.
- Annotated graph showing simulated position vs time for simple harmonic oscillator.