Experiment 2: Springs and Oscillations

2.C. Damped Harmonic Motion

This week you will continue to study a mass oscillating on a vertical spring but will use masses that are purposefully large so that they experience a non-negligible damping force due to air drag.

Background

As you have learned in class, an object moving through a medium such as air experiences a resistive, or damping, force that varies with the object's speed. The exact nature of this force depends on several factors such as the size and shape of the object, the density and viscosity of the resistive medium, and whether the object is moving slowly or quickly. Most situations can be described with a force that depends on the speed in either a linear or quadratic manner, and is in the opposite direction to the velocity:

Linear Damping	Quadratic Damping
$\vec{F}_r = -b\vec{v} = -bv\hat{v}$	$\vec{F}_r = -cv^2\hat{v}$

The constants b and c are called *damping coefficients* and depend upon the details of the object's shape and the properties (e.g., density, viscosity, …) of the resistive medium. At extremes of speed or size, usually one or other of these terms dominates and it is adequate to describe the force with only that term. For example, for a freefalling skydiver the quadratic formula is most appropriate. However, in the situations we will study in this laboratory, the speeds are small enough that the linear formula is the most accurate representation.

Finding analytical solutions for situations where objects experience these kinds of forces requires the use of differential equations, which is beyond the scope of this course. It can be shown using differential equations that the motion of a one-dimensional oscillator of mass m that experiences both a restoring (spring) force $-kx$ and a linear damping force $-bv$ can be described by

$$x(t) = A\, e^{-\gamma t} \cos(\omega t + \phi)\ ,$$

where x is the amount the spring is stretched, and the constants A and ϕ (phase) depend on the initial position and velocity of the oscillator. The *damping parameter* γ (s^{-1}) is related to the damping coefficient b by

$$\gamma = b/2m$$

and the oscillation frequency ω depends on the damping parameter through the relation

$$\omega = \sqrt{\omega_0^2 - \gamma^2}\ ,$$

where $\omega_0 = \sqrt{k/m}$ is the frequency of the undamped oscillator. So the damping causes both the amplitude to steadily decrease with time (the $e^{-\gamma t}$ factor) and the oscillation frequency to be (usually slightly) less from what it would be without damping.

Interestingly, there are forms of the damping force where analytical solutions cannot be determined, and therefore a computational treatment, such as you will explore with VPython, is needed to describe the motion of the object. Substantial confidence in the power of such numerical treatments can be gained by comparing such solutions to the corresponding analytic solutions when they exist, as is the case here.

Equipment/supplies provided:

- Sonic ranger, interface box, and computer.
- Spring, set of 5 masses with different sizes.
- Reinforced pole for hanging spring from.

Students are responsible to have:

- Laboratory manual with worksheet pages for recording observations and data.

Experimental Tasks:

1. ***Measure the frequency and damping coefficient for a set of different size oscillators.***

   1.1. Find the same spring that you used last week. If you cannot locate this spring, you will need to obtain a new one and determine its spring constant. Spring A

   1.2. You will be provided with a set of five different size disks that have similar (not necessarily exactly the same) masses. Select the smallest disk and measure (and record) its mass and radius.

   1.3. Set up Lab Assistant and the sonic ranger to measure the position of this mass as it oscillates vertically when attached to your spring.

   1.4. Set the system into motion, let any side to side oscillation subside, and then collect a set of smooth data for at least a 10 second interval. see E2C-mass1.txt

   1.5. When you have a good set of data, save it to a tab-delimited text file and load it into Physics Data Assistant to create a plot that shows the position of the oscillator as a function of time.

   1.6. Perform a best fit to this data using the underdamped oscillator model described by

   $$y(t) = y_0 + A\, e^{-\gamma t} \cos\left(\omega t + \phi\right)$$

   and determine values for the fit parameters y_0, A, γ, ω, and ϕ. Record these values in your laboratory worksheet. Annotate the damping coefficient and angular frequency on your plot and export a copy for your report. see files E2C-mass2.txt thru E2C-mass5.txt

   1.7. Repeat this process for the other four disks.

   1.8. Combine your results into a table that shows for each disk the radius r, the damping coefficient b (remember that $b = 2\gamma m$), and the oscillation frequency ω.

   1.9. Use your data to determine a relationship between the damping coefficient b and the radius r of the disk. Create a plot and use it to determine a fit function that allows you predict a damping coefficient for a given disk radius. (<u>*Hint:*</u> Try plotting the damping coefficient versus the radius and versus the area to see which model works best. Does the result you find make sense?)

   1.10. Describe how the frequency of the oscillator changes (or doesn't) as the damping changes.

2. ***Modify your VPython oscillator model to include damping.***

   2.1. Duplicate your VPython program for the simple harmonic oscillator from last week.

   2.2. Modify this model by adding a linear damping force of the form $F_r = -bv = -2m\gamma\, v$. Of course, this force will need to be inside the computation loop since it will change on each iteration as the velocity of the oscillator changes. Be careful to correctly model the direction of this force.

Set up your program so that the value of the damping coefficient b is calculated from the disk radius based on the relationship you found in paragraph 1.9 above. Try different values of the radius and see how well your model reproduces your experimental data for a disk of that radius.

For Next Week

- Considering your work from the past three weeks on springs and oscillators as a whole, identify any problems or issues with your data or models that you will want to revisit next week.
- Prepare a draft report that includes an introduction, experimental description, data and analysis, modeling, and conclusion sections. Bring two printed copies to class to do a peer review.