Virtual Physics

Virtual Physics is a software package that allows one to numerically solve a set of first-order differential equations. It is most commonly used is to solve for the position and velocity coordinates of a system if the forces acting on the system are known.

To illustrate its use we start by considering a one-dimensional system consisting of a single particle of mass m. Newton's Second law for the particle can be expressed as

$$F(x, v_x; t) = m\frac{d^2x}{dt^2}$$

where $F(x,\ v_x,\ t)$ is the net force acting on the particle which may depend upon the position of the particle x, the velocity of the particle v_x, or even explicitly on the time t. This expression is a second-order differential equation since the force depends upon the acceleration of the particle, which we write as the second derivative of the position. In this equation time is the independent variable. The position $x(t)$ and velocity $v_x(t)$ are unknown functions that depend upon time. When we solve this differential equation we hope to find $x(t)$ and $v_x(t)$. The solution will depend upon the details of the net force as well as the initial values of position $x(0)=x_0$ and $v_x(0)=v_{x0}$.

In order to use Virtual Physics the equation(s) of motion must be re-written as a system of first-order differential equations. For this particle we can take the second-order differential equation expressed above as Newton's Second Law and rewrite it as two first-order differential equations. This is accomplished by noting that the acceleration is the derivative of the velocity and the velocity is the derivative of the position.

$$\frac{dv_x}{dt} = \frac{F(x, v_x; t)}{m}$$

$$\frac{dx}{dt} = v_x$$

Now we have two differential equations both of which are solved for the derivatives of the unknown functions.

Now, to make this example more specific, let's assume our system is a mass m connected to a spring of force constant k. Further, let's suppose that the mass moves through a resistive medium where the drag force on the mass is directly proportional to the speed. In this case, the net force on the mass will be given by

$$F = -kx - bv_x$$

where b is a constant that depends on the size and shape of the particle as well as the medium. Then, our equations that express the derivatives of the velocity and position are

$$\frac{dv_x}{dt} = -\left(\frac{k}{m}\right)x - \left(\frac{b}{m}\right)v_x$$

$$\frac{dx}{dt} = v_x$$

These expressions are then entered into the Differential Equations tab on Virtual Physics as shown below.

Description	Variable	Differential Equation	Initial Condition	
Velocity	vx	-(k/m)*x-(b/m)*vx	0	
Position	x	vx	2	

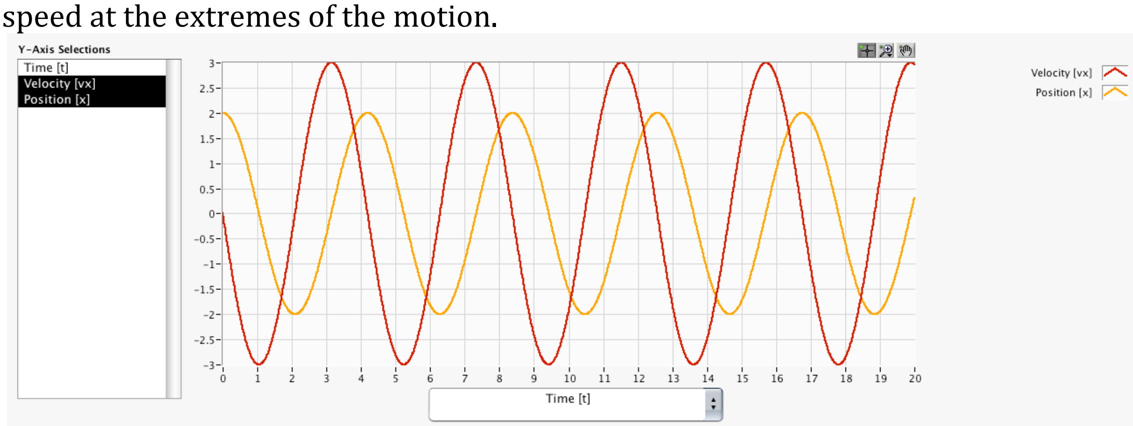
The Description column simply contains the name of the unknown variable while the Variable column contains its symbolic representation. The differential equation column contains a function that equals the derivative of the associated variable. Also, on this table one must enter numerical values for the initial conditions of each variable. In the example above the initial velocity is set to 0 m/s and the initial position is 2 m.

At this point, the program will not know values for any constants in the differential equations so it will not be able to solve for the velocity and position functions. Values of any constants must be entered on the calculations tab.

Description	Name	Value	
Spring Constant	k	18	
Mass	m	8	
Drag Coeffiecient	b	0	

The symbolic representation of the constants must match exactly the form that is used in the differential equations. Numerical values for each constant must be entered into the value column. For our example, we have set the spring constant to 18 N/m, the mass to 8 kg, and for now have set the drag coefficient to zero.

After entering the differential equations and the values for any constants the software should be able to solve the system of equations, subject to the initial conditions, to give the velocity and position coordinates as a function of time. If there are no errors then you should see an "Input OK" message at the bottom of the Differential Equations tab. The results for the Velocity and or Position can be viewed either on the Graph or on the Table on the top of the screen. To view a graph of the velocity as a function of time click on the Graph tab and then click Velocity on the Y-Axis selections. Multiple variables can be selected by holding the shift key while clicking. Notice in the graph below how the particle has its maximum speed when the spring isn't compressed or stretched (x=0) and has zero speed at the extremes of the motion.

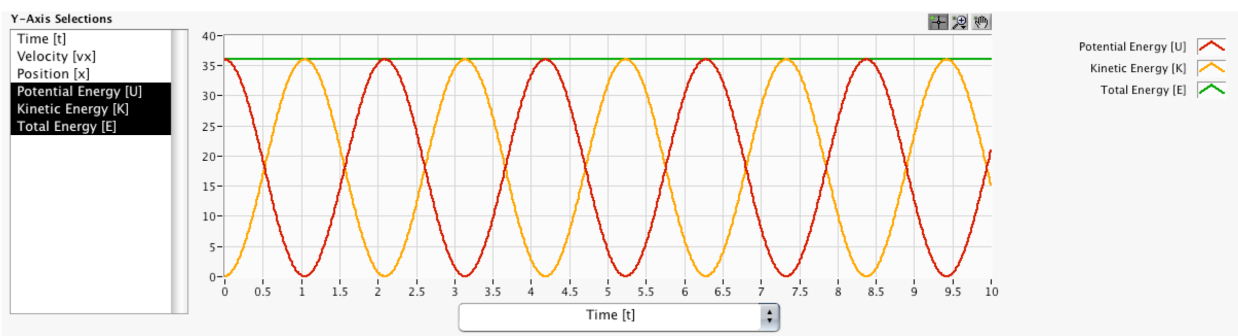


The range of the independent variable is controlled on the Integration Setup tab. On this tab you can set the symbol and name of the independent variable (usually t and Time) as well as setting the starting value, ending value, and increment for the independent variable. The default values are from 0 to 10 seconds in increments of 0.01 seconds. This will result in a table of 1000 data points. Also, the algorithm used to solve the system of equations can also be set on this tab. The most accurate results are usually obtained using the Runge-Kutta method.

In addition, one can create additional functions that are calculated from the functions that are solved for by Virtual Physics. A common use is to compute potential and kinetic energies from the position and velocity coordinates. For example, for the spring problem we have been discussing one can calculate the elastic potential energy, kinetic energy, and total energy of the particle by entering the following on the calculations tab.

Description	Variable	Formula	
Potential Energy	U	(1/2)*k*x^2	
Kinetic Energy	K	(1/2)*m*vx^2	
Total Energy	E	K+U	

These quantities, which are calculated from the solved functions, are added to the list of functions available on the Y-Axis Selections. One can create a graph that shows the kinetic energy, potential energy, and total energy as a function time. Notice how as time progresses the energy is changed between kinetic and potential forms but the total energy remains constant.



Or, one can change the selection on the horizontal axis to graph the potential energy, kinetic energy, and total energy as a function of position. This graph allows one to see where the particle has all kinetic energy (moving the fastest at x=0) or all potential energy (zero velocity at x=±2), but of course the total energy still remains constant.

Finally, one can also investigate how the motion changes as various parameters are changed. For example, if we increase the damping coefficient we can see the effect of the resistive force on the motion. The graphs below show the velocity and position as a function of time and then the energies as a function of time when the damping coefficient has increased to 4 Ns/m. In particular notice that the total energy is no longer conserved.