

Magnetic Field of a Solenoid

Purpose

The purposes of this experiment are:

1. to measure the magnetic field in and around a long solenoid,
 2. to measure the magnetic field inside a solenoid as a function of current, and
 3. to determine the permeability of free space using the experimental data and theory.
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Introduction

A solenoid is a long helical coil of wire through which a current is run in order to create a magnetic field. The magnetic field of the solenoid is the superposition of the fields due to the current through each coil. It is nearly uniform inside the solenoid and close to zero outside and is similar to the field of a bar magnet having a north pole at one end and a south pole at the other depending upon the direction of current flow.

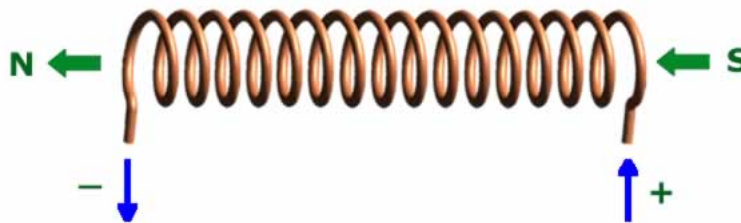


Figure 1

In this experiment you will use a magnetic field sensor to measure the magnitude and direction of the magnetic field of a solenoid. A Slinky will be used to create the solenoid. The magnetic field sensor will be placed thru the coils of the Slinky to measure the field. Care must be taken to ensure that your measurements do not include the field associated with the Earth or any other nearby objects.

Theory

To prove the above statements about the field inside and outside a solenoid we will first use some symmetry arguments to determine the direction of the field and then use Ampere's Law to find the field magnitude. We will consider an *ideal* solenoid, one that has the radius of its coils very small compared to its overall length and has its coils closely spaced together. A perfectly ideal solenoid would have infinite length.

To understand the solenoid field, we will first start with a straight wire, then a single coil, then multiple coils. Magnetic field lines always form loops around the current. The direction of the field can be found by the right-hand grip rule. For a single long straight wire the field direction is given by placing the thumb of the right hand in the direction of the current, then the fingers will curve in the direction of the field as shown in Figure 2.

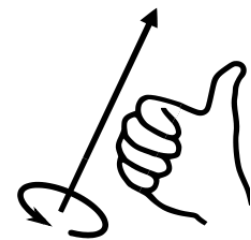


Figure 2

Now if you imagine bending the wire into a circle and apply the right-hand grip rule to a single coil of the solenoid, the field lines still wrap around the current in the wire. Figure 3 shows an edge on

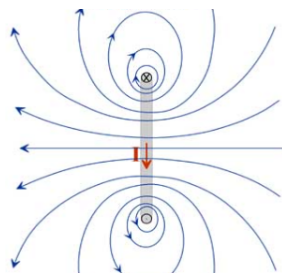


Figure 3

When you superimpose the fields for several adjacent coils of an ideal solenoid the components of the field inside the solenoid that are not exactly to the left cancel out leaving a purely leftward pointing field as shown in Figure 4. On the outside a similar cancellation occurs and the field points to the right. The field is concentrated in the interior of the coil and is considerably weaker outside the coil. The longer the coil the weaker the field outside the coil will be. An infinitely long coil will have zero field outside.

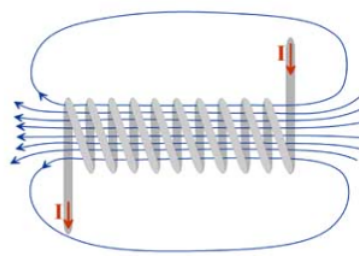


Figure 4

Now, we apply Ampere's Law to determine the value of the magnitude of the field. Ampere's Law states that the line integral of the magnetic field around a closed path P is proportional to the amount of current that is enclosed by the path.

$$\oint_P \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} . \quad (1)$$

The constant of proportionality is called the permeability of free space and has a value of

$$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A} \quad (2)$$

where T is the SI unit of magnetic field strength. The fields that you will measure in this experiment are much smaller than a Tesla so we will use units of Gauss when measuring field strength. The conversion between Tesla and Gauss is given by

$$1 \text{ G} = 10^{-4} \text{ T} . \quad (3)$$

To apply Ampere's Law consider an imaginary Amperian loop in the shape of a rectangle $abcd$ as shown in Figure 5. On path ab the field is in the same direction as $d\vec{l}$ so the integrand will just have the value BL . On paths bc and da the field will be perpendicular to the path and $\vec{B} \cdot d\vec{l}$ will be zero. Path cd can be chosen to be a very large distance from the solenoid where the field will be zero. When these observations are inserted into Ampere's Law we will obtain

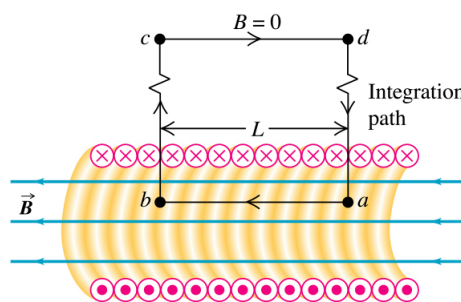


Figure 5

$$\oint_P \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \quad (4)$$

$$\oint_P \vec{B} \cdot d\vec{l} = BL + 0 + 0 + 0 = \mu_0 I_{enc} \quad (5)$$

The current enclosed in the loop will be the number of turns N in the length L that go thru the loop

multiplied by the current I in each coil $I_{enc} = NI$. So, equation (5) simplifies to

$$BL = \mu_0 NI . \quad (6)$$

If we define the number of turns per unit length as $n = N/L$, then we have the result for the field magnitude inside the solenoid

$$B = \mu_0 nI . \quad (7)$$

This equation gives the field inside an infinitely long solenoid whose coils are very closely spaced. You will have the opportunity in this experiment to see how well this equation holds for a solenoid that is not ideal. In particular you will measure the field at the end and outside on the solenoid axis.

Apparatus

You will create a solenoid using a metal Slinky and will measure the magnetic field using an iWorx magnetic field sensor MGN-100. A power supply that can supply 6 volts and 1 amp will be required to supply a current for the solenoid. Clearly the Slinky will not create an ideal solenoid so you will be able to see how well the field is described by the model $B = \mu_0 nI$.

The magnetic field sensor uses a Hall effect transducer that produces a voltage that is linear with the magnetic field. The Hall effect transducer is located at the end of the sensor in the little tab that protrudes past the heat shrink tubing. It measures the component of the magnetic field that is perpendicular to the flat area of the tab.

The sensor will output a voltage near 2.5 V when zero magnetic field is present. The output will increase or decrease depending upon the direction of the sensed magnetic field. The scaling between magnetic field and voltage is 200G per volt. The minimum and maximum fields measureable by the sensor are -420G and +420G corresponding respectively to 0.4V and 4.6V.

The range of the sensor is too large for the fields encountered in this experiment. An instrumentation amplifier is used to amplify the sensor output. The instrumentation amplifier is an INA129 that uses an external resistor of 100 Ω to have a gain of 500. The output from the sensor is directed to the one input of the amplifier and a voltage divider network consisting of two 11k Ω resistors and a 2 k Ω trim pot in the center allow the other input to be adjusted between 2.29V and 2.71V. The amplifier will output the difference in these two voltages multiplied by a factor of 500. The trim pot then allows a method for adjusting the “zero-field” output of the sensor to zero volts. For our purposes zero-field means the field with the current to the solenoid turned off. The trim adjustment will allow the field of the Earth and other nearby objects to be nulled out of the measurement.

INSERT DIAGRAM OF AMPLIFIER CIRCUIT

The current in the circuit can be read directly from the power supply if available or a shunt resistor of known resistance (less than 1 Ω) can be placed in series with the solenoid. The voltage across the shunt can be measured with one of the analog inputs on the myDAQ and the current can be calculated from Ohm’s law.

INSERT DIAGRAM OF SOLENOID and CURRENT SHUNT

Pre-Lab Questions

1. If you look up the permeability constant in a reference, you may find it listed in units of henry/meter. Show that these units are the same as tesla-meter/ampere.

2. Consider the magnetic field at the end of a solenoid on its axis. Use a symmetry argument to predict what the value of the field will be at this point?

HINT: Consider a point P on the interior of an infinitely long solenoid. The field at that point will be equal to a superposition of the field due to the coils on the left of P and the field due to the coils on the right of P . How will these two contributions compare to each other. What will happen to the field if the coils on the right of point P are removed?

3. Make a guess as to the number of coils in a Slinky. Using this guess compute the magnitude of the magnetic field that you would expect on the interior of the solenoid if you stretch the Slinky to a length of 1 m and apply a current of 1.5 A. How does this value compare to the typical value of the magnitude of the Earth's magnetic field at its surface?

4. Would you expect different results if your solenoid is oriented on a North-South, East-West, or other axis?

In-Lab Procedure

Setup

1. Connect the iWorx MGN-100 magnetic field sensor to the amplifier circuit and connect the output of the circuit to one of the analog inputs of the myDAQ (or ELVIS). Be sure to record in your lab notebook the analog input channels you use for the light sensor.
2. Connect the myDAQ (or ELVIS workstation) to the computer with the USB cable.
3. Open the *Physics Lab Assistant* software. Create a waveform called Magnetic Field using the Add button on the Analog Input Waveforms tab. Be sure to associate this waveform with the same physical channel that you connected the magnetic field sensor to previously. The raw output of the sensor is in volts with a gain of 200 G/V. The instrumentation amplifier amplifies this signal by a factor of 500 so what we measure is actually 500 times smaller. As a result, the gain should be 200 G/V divided by 500 or 0.4 G/V. When defining the channel set the Gain to 0.4, the Offset to 0, and the Units to G.
4. Use the Check button to check the output of the sensor. You may need to rescale the axis of the graph to suitable values by clicking on the upper and lower limits of the vertical axis and entering new values. Notice that the output of sensor is very sensitive and is dependent on its position and orientation in space. The magnetic field that the sensor measures is the component of the field that is perpendicular to the flat area on the end of the sensor. Use a permanent magnet to test the sensor operation.

Measurement of Earth's Magnetic Field

5. Hold the probe away from any possible stray magnetic fields and any metal objects. Attempt to measure the Earth's magnetic field at a specific location by orienting the probe for a maximum reading and then reversing the probe for a minimum reading. The Earth's field at that location will be half of the difference between these two readings and the direction of the field at that location will be perpendicular to the tab on the end of the sensor.
6. If you hold the probe vertically and rotate it for maximum reading you should be able to determine the direction of magnetic north. Once you have located magnetic north you can rotate the sensor downward until the reading is maximum again and this will allow you to determine the magnetic inclination (angle made by the local magnetic field with the horizontal). See if you can determine the direction of magnetic north and the magnetic inclination at your workstation. Your results will likely be strongly influenced by your surroundings including the metal in the building.

Measurement of the Field of a Solenoid

7. Arrange the Slinky on the lab table in a straight line with the coils separated far enough apart to be able to easily insert the magnetic field sensor. Clamp the ends in place so that the Slinky will not move.
8. Count the number of turns of the Slinky and record this value as N . Measure the length L of the Slinky and then compute the number of turns per unit length n .
9. Connect the power supply to a series circuit consisting of the shunt resistor and the Slinky solenoid. If available insert a knife-edge switch into the circuit so that you can easily turn the current to the solenoid off and on.

10. Connect a pair of wires across the shunt resistor in a parallel fashion to an analog input on your myDAQ (or ELVIS).
11. Using the Add button on the Analog Input Waveforms tab define a channel for the current in the circuit. You will measure the voltage across the shunt and calculate the current using Ohm's law. The current will be the measured voltage divided by the resistance. You can use the Gain when defining this channel to automatically convert your measured voltage to a current. For example, if your shunt resistance is $1/3\ \Omega$ then to compute current you would divide by $1/3$ or multiply by 3. So in this example a gain of 3 would be appropriate and then the signal would be scaled to measure the current using Units of Amperes (A).
12. Using the Timing button on the Analog Input Waveforms tab set the timing to measure for a Total Time of 5 seconds, set the Buffer to 0.200 so that the screen will refresh 5 times every second, and set the AI Period to 0.001 seconds so that the channels will be measured 1000 times a second.
13. In the Calculated Values tab set up calculations to measure the mean and standard deviation of both the magnetic field and the current. You should have four calculations total.
14. Turn on the power supply and set the output to 0 V. Use the Verify button on the Analog Input Waveforms tab to monitor the current. Slowly increase the voltage of the power supply and monitor the current. Do not exceed 1.5 A. As you adjust the voltage output of the power supply the current should increase to a value determined by the resistance of the circuit (Slinky plus shunt), the voltage setting, and Ohm's law. Monitor the current reading on from your data acquisition system and make sure that it matches closely the value displayed on the power supply.
15. Use your magnetic field sensor to survey the field in and around the solenoid. Test the center of the Slinky noting the orientation of the field and the uniformity (or lack thereof) at different radii. Test different points along the length of the solenoid. Test the end of the solenoid and then move along the axis outside the solenoid to see how rapidly the field strength drops off. Finally measure the field outside the solenoid. You can use the switch to turn the current on and off to make sure your observations are due to the current in the solenoid and not some other stray field. Make enough observations that you can qualitatively describe the nature of the field inside and around the solenoid and compare your observations to the theory.
16. When making the measurements that follow it will be very important to zero the output of the sensor in between each measurement. The Hall effect sensor has a slight tendency for the zero-field value to drift. Locate the sensor in the center of the solenoid oriented for maximum output. Do not disturb the sensor or the solenoid for any of the remaining measurements. In between each measurement you will need to turn the current off and set the output of the sensor to zero to null out the effect of the Earth's field and any other stray fields. Then you can turn on the current, hit Acquire to make a measurement at the set current value, and then turn the current back off. This will make for a tedious data collection process but the results will pay off in the quality of the data.
17. Set the output of the power supply to give a current of approximately 0.1 A, but then turn off the current using the switch. Zero the output of the magnetic field sensor. Turn on the current and then use the Acquire button to collect a set of measurements of the magnetic field and the current. If the data is as expected then hit the Calculate Now button to compute the average and standard deviation of the magnetic field and the current and display the results in the table.



18. Repeat the above measurement for currents at intervals of 0.1 A up to a maximum of 1.5A.
19. Save the data in the Calculations table to Excel, Igor Pro, or tab-delimited text format depending upon your scientific graphing software.
20. Create a graph of magnetic field strength versus current. Perform a linear best fit to the data and record the slope. Using your value for the number of coils per unit length of the solenoid and the slope calculate an experimental value for the permeability of free space, μ_0 .
21. Compare your measured value for the permeability of free space to the theoretical value of $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$ by computing a percent discrepancy. Be careful with unit conversions for the magnetic field. Since your sensor measures in Gauss you will need to perform a conversion from Gauss to Tesla.

Post-Lab

1. Discuss your measurements of the Earth's magnetic field. Was this process easy or difficult?
2. Discuss your measurements of the field inside and around the solenoid in a qualitative manner. Did the direction of the field inside the solenoid agree with what you expected? Was the field uniform inside the solenoid? What was the field at the end of the solenoid in relation to the value in the interior? Did this agree with what you predicted in the pre-lab question? How quickly did the field fall off as you moves away from the solenoid along the axis?
3. Discuss your measurements of the magnetic field of the solenoid as a function of current. Did these data agree with the theory? What value of the permeability of free space did you obtain and how well did it agree with the published value?